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I. Solution by the PROPOSER.

It can be demonstrated that D , in $x^2 - Ay = \pm D$, can be any denominator of the complete quotients from the \sqrt{A} , and that x and y are the numerator and denominator of the convergent preceding the term in which D is taken. Now the complete quotients for the $\sqrt{114\frac{1}{4}}$ are

$$\frac{0 + \sqrt{114\frac{1}{4}}}{1}; \quad \frac{10\frac{1}{2} + \sqrt{114\frac{1}{4}}}{4}; \quad \frac{9\frac{1}{2} + \sqrt{114\frac{1}{4}}}{6}; \quad \frac{8\frac{1}{2} + \sqrt{114\frac{1}{4}}}{7}; \text{ etc.}$$

No. term 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, etc., reversing.

$\therefore \sqrt{114\frac{1}{4}} = \frac{1}{6}, \frac{10\frac{1}{2}}{1} : 5, 3, 2, 1, 2, 1, 6, 2, 1, 1, 10, 10, \text{ etc.}, \text{ reversing.}$

Complete denom'rs = 1 : 4, 6, 7, 12, 6, 14, 3, 8, 9, 12, 2, 2, etc., reversing.

Hence x and y are found in the 6th convergent and also the 15th convergent, and $x = 2095$ and 196 , and also from the 15th term $x = 42, 307, 834$, and $y = 3, 958, 154$. [Also see problem 38.]

II. Solution by JOSIAH H. DRUMMOND, LL. D., Portland Maine.

I have not solved this problem as stated, but as I have solved it in this form $x^2 - 114\frac{1}{4}y^2 \pm 3 = \square \dots \dots \dots (1)$, and as that is a pretty question, I send my solution.

Multiplying by 4, it becomes $4x^2 - 457y^2 \pm 12 = \square = \text{say } (2x - m^2 = 4x^2 - 4mx + m^2)$, from which we find

$$x = \frac{457y^2 + m^2 \pm 12}{4m}.$$

$(m \pm 12)/4m$ evidently becomes integral when $m = 6$; and we have

$$x = \frac{457y^2}{24} + 2, \text{ or } \frac{457y^2}{24} + 1.$$

$457y^2/24$ becomes integral when $y = 12n$, and $x = 2742n + 2$, or $= 2743n + 1$, according as the + or - sign before 3 is taken.

If $n = 1$, $y = 12$, and $x = 2744$ or 2743 ; in the former, 3 is negative, and in the latter, positive, in order to make the expression a square.

54. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

In the expression $2x^2 - 2ax + b^2$, find two series of values for x in integral terms of a and b .

I. Solution by the PROPOSER.

$2x^2 - 2ax + b^2$ is evidently a square when $x = a$. Take $x = y + a$, and substituting, we have $2y^2 + 2ay + b^2 = \square = (\text{say})(my - b)^2$.

Reducing, $y = 2(a + bm)/(m^2 - 2)$. Taking $m = 2/1, 10/7, 58/41$, etc., we have one integral series of the value of y , viz.: $a + 2b, 49a + 10b$, etc. Taking $m = 3/2, 17/12, 99/70$, etc., we have another integral series of the value of y , viz.:

$8a+12b$, $288a+408b$, etc. By adding a to each term of each series we have two series of the value of x . These series hold good when either a or b is zero; but if both are zero, $x=0$.

It will be noticed that this solution applies the terms of the question to the expression $2x^2+2ax+b=\square$, the value of x in the latter being a less than in the former.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

$$2x^2-2ax+b^2=0. \therefore x=\frac{1}{2}(a\pm\sqrt{a^2-2b^2}).$$

Let $a=p^2+2q^2$, $b=2pq$. Then $x=p^2$ or $2q^2$.

\therefore	p	q	a	b	x ,
	2	1	6	4	4 or 2,
	3	2	17	12	9 or 8,
	4	3	34	24	16 or 18,
	1	2	9	4	1 or 8,
etc.	etc.	etc.	etc.	etc.	etc.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

54. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

A man is at the center of a circle whose diameter is equal to three of his steps. If each step is taken in a perfectly random direction, what is the probability, (1), that he will step outside the circle at the second step, and, (2), that he will step outside at the third step?

I. Solution by the PROPOSER.

Let O be the center of the circle, A , the end of the first step, and B , the end of the second, and C , the end of the third.

Let $\angle OAB=\theta$, $\angle OBC=\phi$, $OB=x$, and $OC=y$.

Then if the length of the step be taken as the unit of measure, $x=2\sin\frac{1}{2}\theta$, and $y=(x^2+1-2x\cos\phi)^{\frac{1}{2}}=(4\sin^2\frac{1}{2}\theta+1-4\sin\frac{1}{2}\theta\cos\phi)^{\frac{1}{2}}$.

If $x=\frac{3}{2}$, B falls upon the circumference of the circle, and $\theta=2\sin^{-1}\frac{3}{4}$. If $\theta>2\sin^{-1}\frac{3}{4}$, and $\theta<\pi$, the second step falls outside the circle. The probability of this is $P_1=(\pi-2\sin^{-1}\frac{3}{4})/\pi$.

If θ be $<2\sin^{-1}\frac{3}{4}$, and $y=\frac{3}{2}$, C falls upon the circumference of the circle, and $4\sin^2\frac{1}{2}\theta+1-4\sin\frac{1}{2}\theta\cos\phi=\frac{9}{4}$ or $\phi_1=\cos^{-1}(\sin\frac{1}{2}\theta-5/16\sin\frac{1}{2}\theta)$. Hence if ϕ be $>\phi_1$, the third step falls outside the circle. The chance that ϕ will be $>\phi_1$, and $\phi<\pi$ is $(\pi-\phi_1)\pi$. The chance that θ has any particular value is $d\theta/\pi$. Hence the probability that the third step falls outside the circle is